Department of Economics **University of Manitoba**

ECON 7010: Econometrics I

FINAL EXAM, Dec. 17th, 2015

Instructor:	**Ryan Godwin**
Instructions:	**Put all answers in the booklet provided.**
Time Allowed:	**3 hours.**
Number of Pages:	**4**

There are a total of 100 marks.

PART A: Short answer. ***Answer 4 out of 6 questions.*** *Each question is worth 5 marks.*

1.) Describe the basic idea behind the simple bootstrap, and how you could use the simple bootstrap to construct a confidence interval.

2.) Briefly explain why OLS will not work when the dependent variable is a "count" variable.

3.) Show that the following formulation of the F-statistic can be rewritten in terms of R^2 from the restricted and unrestricted models:

$$F = \frac{(e_*'e_* - e'e)/J}{e'e/(n-k)},$$

where e_* are the residuals from the restricted model.

4.)

5.)

6.)

Choose 4 questions from above (and below). I will only mark the first 4 questions in each of PART A and PART B.

PART B: Long answer. ***Answer 4 out of 6 questions.*** Each question is worth 20 marks.

1.) Consider the linear multiple regression model, with k non-random regressors:

$$y = X\beta + \varepsilon \quad ; \quad \varepsilon \sim N[0, \sigma^2 I]$$

Suppose that one of the regressors is a dummy variable, and that this dummy is zero for all but one of the 'n' observations in the sample.

[*Hint*: *OLS does not depend on the order of the data, so make things easier for yourself by assuming that the single 'special' observation is the last one in the sample*.]

a) Prove that the OLS estimators for the coefficients of the other regressors in the model are the same as would be obtained if the dummy variable were omitted from the model, and the one observation for which the dummy is non-zero was also dropped from the sample.

[15 marks]

b) Prove that the last element in the residual vector is zero.

[5 marks]

2.)
3.) An AR(1) model for the error process is:

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t \quad ; \quad u_t \sim i.i.d.N[0\,, \sigma_u^2] \quad ; \quad |\rho| < 1,$$

and an MA(1) process for the error term is:

$$\varepsilon_t = u_t + \phi u_{t-1} \quad ; \quad u_t \sim i.i.d.N[0\,, \sigma_u^2].$$

a) Prove that an AR(1) process has "infinite memory".

b) Derive the variance of ε_t in an AR(1) model. What condition is required for this variance to be finite?

c) Describe how you could estimate ρ from an AR(1) model.

d) Suppose that the error term is AR(1). Suppose that the y and X data are transformed such that:

$$y^* = \begin{bmatrix} y_1\sqrt{1-\rho^2} \\ y_2 - \rho y_1 \\ \vdots \\ \vdots \\ y_n - \rho y_{n-1} \end{bmatrix} \quad ; \quad x_j^* = \begin{bmatrix} x_{1j}\sqrt{1-\rho^2} \\ x_{2j} - \rho x_{1j} \\ \vdots \\ \vdots \\ x_{nj} - \rho x_{n-1,j} \end{bmatrix} \quad ; \quad j = 1, 2, \ldots, k$$

Verify that the error term from this model (ε^*) is not autocorrelated.

[5 marks each]

4.)

a) What are some of the issues associated with OLS estimation, in the presence of heteroskedasticity?

b) Describe White's test for heteroskedasticity, and how you would perform it.

c) Describe the Goldfeld-Quandt test for heteroskedasticity, and how you would perform it.

d) Suppose you reject the null hypothesis in a Goldfeld-Quandt test. Describe how you would implement FGLS.

[5 marks each]

5.) When we establish the (weak) consistency of the OLS estimator for the regression coefficient vector, we usually assume that

$$plim\left(\frac{X'X}{n}\right) = Q \quad ; \quad Q \text{ is a finite, positive-definite matrix.}$$

Suppose that we have a model with just <u>two</u> regressors – an intercept and a linear time-trend variable (a variable that takes the values 1, 2, 3,….., *n*).

a) Is Q finite and positive-definite in this case?

b) Obtain the expressions for the mean and variance of the OLS slope coefficient estimator for this special model.

c) Use these expressions to prove that this OLS estimator is 'mean square consistent'.

d) Is this estimator 'weakly consistent'?

[5 marks each]

6.)

END.